Chapter 4 (Determinants)
Important Questions Unsolved

SECTION - A

Q.1. If A_{ij} is the cofactor of the a_{ij} of the determinant.

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$ the value of $a_{32} A_{32}$.

Q.2. If A is a 3×3 invertible matrix, then what will be the value of k if

$\det(A^{-1}) = (\det A)^k$.

Q.3. If A_{ij} is the cofactor of the a_{ij} of the determinant.

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$ the value of $a_{32} A_{32}$.

Q. 4. Evaluate: $\begin{vmatrix} a + ib & c + id \\ -c + id & a - ib \end{vmatrix}$

Q. 5. Find the co-factor of a_{12} in the following:

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

Q.6. Write the value of:

$$\Delta = \begin{vmatrix} x + y & y + z & z + x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$$

Q.7. If $x \in N$ and $\begin{vmatrix} x + 3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x.

Q.8. Write the value of the determinant:

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$$

Q.9. If A is an invertible matrix of order 3 and $|A| = 5$, then find $|adj.\, A|$.

Q. 10. What is the value of the determinant:

$$\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}?$$

Q.11. If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the minor of the element a_{23}.

Q. 12. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x.

Q. 13. Write the element a_{23} of a 3×3 matrix $A = (a_{ij})$ whose elements a_{ij} are given by: $a_{ij} = \dfrac{|i - j|}{2}$.

Q.14. Find the maximum value of

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 + \sin\theta & 1 \\ 1 & 1 & 1 + \cos\theta \end{vmatrix}$$

Q15: If for any 2×2 square matrix A, $A(adj\, A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$.

Q.16. If A is an invertible matrix of order 3 and $|A| = 5$, then find $|adj.\, A|$.

Q.17. Write the value of the following determinant:

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}.$$

Q.18. Find the value of x from the following:

$$\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0.$$

Q.19. A is a square matrix of order 3 and $|A| = 7$. Write the value of $|adj\, A|$.

Q.20. Evaluate: $\begin{vmatrix} \cos 15^o & \sin 15^o \\ \sin 75^o & \cos 75^o \end{vmatrix}$.

Q21. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, **write** A^{-1} **in terms of A.**

Q.22. Let A be a square matrix of order 3 x 3. Write the value of $|2A|$, **where** $|A|=4$.

Q.23. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, **find the value of** x.

SECTION - B

Q.24. A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received Rs 2,800 as interest. However, if trust had interchanged money in bonds, they would have got Rs 100 as interest. Using matrix method find the amount invested by the trust. Interest received on this amount will be given to Helpage India as donation. Which value is reflected in this questions?

Q.25. Using properties of determinants, prove the following:

$$\begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 & c^2 \end{vmatrix} = 4a^2 b^2 c^2$$

Q.26. Find the adjoint of the matrix

$$A = \begin{pmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$$

$A.\,(\text{adj } A) = |A|I_3.$

Q.27. Using properties of determinants, prove that:

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$$

Q.28. Using properties of determinants, prove the following:

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y).$$

Q.29. Using properties of determinates, show that

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4\,abc$$

Q.30. Using properties of determinants, prove the following:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1.$$

Q.31. Using properties of determinants, prove the following:

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2.$$

Q32: Using properties of determinants, prove that

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$$

Q. 33. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, using properties of determinants, find the value of

$f(2x) - f(x)$.

Q.34: Using properties of determinants, prove that

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y)$$

Q.35. By using properties of determinants, prove the following:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1.$$

Q.36. Using properties of determinants, prove that:

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4\,a^2 b^2 c^2.$$

Q. 37. Using properties of determinants, prove that:

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

Q. 38. Using properties of determinants, prove the following:

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2.$$

Q. 39. Using properties of determinants, prove that

$$\begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix} = (x+y+z)^3$$

Q.40. Using properties of determinants, solve the following for x:

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0.$$

SECTION - D

Q. 41. Using matrices, solve the following system of equations:

$x + y + z = 6$
$x + 2z = 7$
$3x + y + z = 12$.

Q.42: If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, Find A^{-1}. Use it to solve the system of equations.

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$.

Q43. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

$x + 3z = 9, -x + 2y - 2z = 4, \quad 2x - 3y + 4z = -3$.

Q.44. using properties or determinants, show that $\triangle ABC$ is isosceles if:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 + \cos A & 1 + \cos B & 1 + \cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$$

Q.45. A shopkeeper has 3 varieties of pens 'A', 'B' and 'C'. Meenu purchased 1 pen of each variety for a total of Rs 21. Jeevan purchased 4 pens of 'A' variety, 3 pens of 'B' variety and 2 pens of 'C' variety for Rs 60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for Rs 70. Using matrix method, find cost of each variety of pen.

Q.46. Using properties of determinants, prove the following:

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta + \gamma & \gamma + \alpha & \alpha + \beta \end{vmatrix} = (\alpha - \beta)(\gamma - \alpha)(\alpha - \beta - \gamma).$$

Q.47. Using matrices, solve the following system of equation:

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

Q.48. Using properties of determinants, show the following:

$$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3.$$

Q.49. Using matrix method, solve the following system of equations:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \qquad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1,$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2; x, y, z, \neq 0$$

Q.50. Using matrices, solve the following system of linear equations:

$$x - y + 2z = 7$$
$$3x + 4y - 5z = -5$$
$$2x - y + 3z = 12.$$

Q.51. A school wants award its students for the value of honesty, regularity and hard work a total cash award of Rs 6,000. Three times the award money for hard work added to that given for honesty amounts to Rs 11,000. The award money given for Honesty and hard work together is double the one given for regularity Represent the above situation algebraically and find the award money for each value, using matrix method apart from these values namely, Honesty, Regularity and hard work, suggest one more value which the school must include for award.

Q.52. Two schools P and Q want to award their selected students on the values of discipline, politeness and punctuality. The school P wants to award Rs x each, Rs y each and Rs z each for the three respective values to its 3, 2 and 1 students with total award money of Rs 1,000. School Q wants to spend Rs 1,500 to award its 4, 1 and 3 students on the respective values (by giving the same award money for the three values as before). If the total amount of awards for one prize on each value is Rs 600, using matrices, find the award money for each value.

Apart from the above three values, suggest one more value for awards.

Q.53. The monthly incomes of Aryan and Babban are in the ratio 3:4 and their monthly expenditures are in the ratio 5:7 if each saves Rs.15,000 per month, find their monthly incomes using matrix method. This problem reflects which value?

Q.54. Prove that:

$$\begin{vmatrix} yz - x^2 & zx - y^2 & xy - z^2 \\ zx - y^2 & xy - z^2 & yz - x^2 \\ xy - z^2 & yz - x^2 & zx - y^2 \end{vmatrix}$$

is divisible by $(x + y + z)$, and hence find the quotient.

Q. 55: Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the

system of equations $x - y + z = 4,\ x - 2y - 2z = 9,\ 2x + y + 3z = 1.$

Q.56. Using properties of determinants, show the following:

$$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3.$$

Q. 57. Using properties of determinants, prove the following:

$$\begin{vmatrix} x & x^2 & 1+Px^3 \\ y & y^2 & 1+Py^3 \\ z & z^2 & 1+Pz^3 \end{vmatrix} = (1+Pxyz)(x-y)(y-z)(z-x).$$

Q.58. Using matrices, solve the following system of equations:

$$4x + 3y + 2z = 60$$
$$x + 2y + 3z = 45$$
$$6x + 2y + 3z = 70$$

Q.59. Using matrices, solve the following system of equation:

$$2x + 3y + 3z = 5,$$

$$x - 2y + z = -4,$$

$$3x - y - 2z = 3$$

Q. 60. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping other (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards.

Q. 61. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award Rs x each, Rs y each and Rs z each for the three respective values to 3,2 and 1 students respectively with a total award money of Rs 1,600. School B wants to spend rs 2,300 to award its 4,1 and 3 student on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is rs 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award.

Q.62. Using properties or determinants, show that $\triangle ABC$ is isosceles if:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 + \cos A & 1 + \cos B & 1 + \cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$$

Q.63. A shopkeeper has 3 varieties of pens 'A', 'B' and 'C'. Meenu purchased 1 pen of each variety for a total of Rs 21. Jeevan purchased 4 pens of 'A' variety, 3 pens of 'B' variety and 2 pens of 'C' variety for Rs 60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for Rs 70. Using matrix method, find cost of each variety of pen.